Homework 8- MATH 2L03

Winter 2016

1. Use the definition of **area under the curve** to express the area under $f(x) = \sqrt[4]{x}$ from 0 to 16. Don't evaluate the limit.

2. Express the limit as a definite integral on the given interval.

   a) $\lim_{n\to\infty} \sum_{i=1}^{n} x_i \sin x_i \Delta x$, on $[0, \pi]$.

   b) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{x_i}{1+x_i} \Delta x$, on $[1, 5]$.

3. Evaluate the following integrals interpreting them in terms of areas.

   a) $\int_{-1}^{2} |x| dx$ b) $\int_{-3}^{3} (1-2x)dx$

4. If $\int_1^6 f(x)dx = 13$ and $\int_{-3}^{1} f(x)dx = 11$. Find $\int_{-3}^{6} f(x)dx$.

5. If $\int_3^{10} f(x)dx = 9$ and $\int_3^{10} g(x)dx = -3$, find $\int_3^{10} [2f(x) + 6g(x)]dx$.

6. Evaluate the following integrals:

   a) $\int_0^2 (6x^2 - 4x + 5)dx$

   b) $\int_0^2 x(2 + x^5)dx$

   c) $\int_{-2}^{-1} \left(4y^3 + \frac{2}{y^3}\right) dy$

   d) $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta} d\theta$

   e) $\int x \sin(x^2)\, dx$

   f) $\int \sec^2(2\theta)\, d\theta$

   g) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$

   h) $\int_0^2 y^2 \sqrt{1+y^3}\, dy$

   i) $\int_0^{\frac{\pi}{4}} (1 + \tan t)^3 \sec^2 t\, dt$

   j) $\int \frac{\sec^2 u}{3 + \tan u}\, du$

   k) $\int \sin\left(e^{x+2}\right) e^x\, dx$

   l) $\int \frac{e^{1/x^2}}{x^3}\, dx$

   m) $\int e^x \sqrt[5]{9 + e^x}\, dx$

   n) $\int x^6 + 6^x\, dx$

   o) $\int \frac{\log_6 x}{x}\, dx$

   p) $\int e^{\tan\theta} \sec^2\theta\, d\theta$

   q) $\int \frac{\cos(\ln u)}{u}\, du$

7. Verify by differentiation that the formula is correct

   (a) $\int \frac{x}{\sqrt{x^2+1}} dx = \sqrt{x^2+1} + C$ (b) $\int x\cos x dx = x\sin x + \cos x + C$.

8. A population of a certain town is increasing at a rate of $t^2 + 2t + 3$ people per year. Find the increase of population during the next 10 years. If the current population is 3000, what will be the population 10 years later.

9. The marginal cost of a production process is $1.5x^2 - 10x + 100$ (measured in dollars), where x is the number of units produced. The fixed cost i.e. the cost when 0 units are produced, is \$50. Find the cost function and use it to compute the cost of producing 20 units.

10. Use the Fundamental theorem of Calculus to find the derivative of the following functions

a) $g(x) = \int_{35}^{x} \sqrt{t^2+1}\, dt$ b) $h(x) = \int_{0}^{x^4} \cos t\, dt$ c) $p(x) = \int_{x}^{x^2} \sqrt{\sin^2 y + 2}\, dy$

11. Evaluate

$$\int_{-3}^{3} x^4 \tan x\, dx$$

Hint: This is a symmetric integral and $\tan(-x) = -\tan x$.

12. Suppose $f(2) = 2$, $f(4) = 3$, $f'(2) = 5$ and $f'(4) = 3$ and f'' is continuous. Find

$$\int_{2}^{4} x f''(x)\, dx$$

13. *Use substitution followed by integration by parts to evaluate the integral

$$\int \cos\sqrt{\theta}\, d\theta$$

14. If f' is continuous on $[a, b]$, show that

$$2\int_{a}^{b} f(x)f'(x)dx = [f(b)]^2 - [f(a)]^2$$

15. If f is continuous and $\int_0^9 f(x)dx = 4$, find $\int_0^3 uf(u^2)du$

16. Find the average value of the function on the given interval

a) $f(x) = 4x - x^2,\ [0, 4]$ b) $h(t) = \dfrac{3}{(1+t)^2},\ [1, 6]$